

## Cryptography III

### Question 1 *Public-key encryption and digital signatures* ()

Alice and Bob want to communicate over an insecure network using public-key cryptography. They know each other's public key.

- (a) Alice receives a message: Hey Alice, it's Bob. You owe me money. Plz send ASAP.  
The message is encrypted with Alice's public key.

◇ *Question:* Can Alice be sure that this message is from Bob?

**Solution:** No. Alice's public key is public. Anyone can encrypt a message under Alice's public key, not necessarily Bob.

- (b) Bob receives a message: Hey Bob, it's Alice. How many dollars do I owe you?  
The message is digitally signed using Alice's private key.

◇ *Question:* Can Bob be sure that this message is from Alice?

◇ *Question:* How does Bob verify this message?

**Solution:** Yes. Only Alice can create a signature under her key.  
Bob can verify it using Alice's public key.

- (c) Alice receives a response: 10000

The message is encrypted with Alice's public key using El-Gamal encryption. Alice decrypted this successfully, but suddenly remembered that she only owed Bob \$100.

◇ *Question:* Assume Bob would not lie. How did an attacker tamper with the message?

◇ *Question:* What could Bob have additionally sent that would've stopped this attack?

**Solution:** The attacker multiplied  $c_2$  by 100, or multiplied  $c_1 \cdot c'_1, c_2 \cdot c'_2$  where  $c'$  is a valid encryption of 100, or they encrypted an entirely new message.

Bob could attach a signature to his original message.

## Question 2 Confidentiality and integrity

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Alice and Bob want to communicate with confidentiality and integrity. They have:

- Symmetric encryption.
  - Encryption:  $\text{Enc}(k, m)$ .
  - Decryption:  $\text{Dec}(k, c)$ .
- Cryptographic hash function:  $\text{Hash}(m)$ .
- MAC:  $\text{MAC}(k, m)$ .
- Signature:  $\text{Sign}_{sk}(m)$ .

They share a symmetric key  $k$  and know each other's public key.

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We assume these cryptographic tools do not interfere with each other when used in combination; *i.e.*, we can safely use the same key for encryption and MAC.

Alice sends to Bob

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1.  $c = \text{Hash}(\text{Enc}(k, m))$
  2.  $c = c_1, c_2$  : where  $c_1 = \text{Enc}(k, m)$  and  $c_2 = \text{Hash}(\text{Enc}(k, m))$
  3.  $c = c_1, c_2$  : where  $c_1 = \text{Enc}(k, m)$  and  $c_2 = \text{MAC}(k, m)$
  4.  $c = c_1, c_2$  : where  $c_1 = \text{Enc}(k, m)$  and  $c_2 = \text{MAC}(k, \text{Enc}(k, m))$
  5.  $c = \text{Sign}_{sk}(\text{Enc}(k, m))$
  6.  $c = c_1, c_2$  : where  $c_1 = \text{Enc}(k, m)$  and  $c_2 = \text{Enc}(k, \text{Sign}_{sk}(m))$

(a) Which ones of them can Bob decrypt?

- 1     2     3     4     5     6

**Solution:** Bob cannot decrypt Scheme 1 because he cannot invert  $\text{Hash}$ . Similarly, he cannot extract the original message from the signature sent in Scheme 5.

The signature does not include the message.

**In sum:** 2-4, 6.

(b) Consider an eavesdropper Eve, who can see the communication between Alice and Bob.

Which schemes, of those decryptable in (a), also provide *confidentiality* against Eve?

- 1     2     3     4     5     6

**Solution:** Scheme 3 does not provide confidentiality because the MAC is sent in plaintext. For the same message, the MAC is the same, thus leaky.

**In sum:** 2, 4, 6.

- (c) Consider a man-in-the-middle Mallory, who can eavesdrop and modify the communication between Alice and Bob.

Which schemes, of those decryptable in (a), provide *integrity* against Mallory? *i.e.*, Bob can detect any tampering with the message?

1       2       3       4       5       6

**Solution:** Scheme 2 does not provide integrity as Mallory can forge a message by sending Bob ( $c'$ ,  $\text{Hash}(c')$ ).

**In sum:** 3, 4, 6.

- (d) Many of the schemes above are insecure against a *replay attack*.

If Alice and Bob use these schemes to send many messages, and Mallory remembers an encrypted message that Alice sent to Bob, some time later, Mallory can send the exact same encrypted message to Bob, and Bob will believe that Alice sent the message *again*.

How to modify those schemes with confidentiality & integrity to prevent replay attack?

◇ The first scheme providing confidentiality & integrity is Scheme .

The modification is:

◇ The second scheme providing confidentiality & integrity is Scheme .

The modification is:

**Solution:** Add a non-repeating nonce or timestamp in the MAC (Scheme 4) or in the signature (Scheme 6).

**In sum:** In both 4 and 6, we replace message  $m$  with  $\text{timestamp} \parallel m$ .

**Question 3** *Why do RSA signatures need a hash?* ( min)

To generate RSA signatures, Alice first creates a standard RSA key pair:  $(n, e)$  is the RSA public key and  $d$  is the RSA private key, where  $n$  is the RSA modulus. For standard RSA signatures, we typically set  $e$  to a small prime value such as 3; for this problem, let  $e = 3$ .

To generate a **standard** RSA signature  $S$  on a message  $M$ , Alice computes  $S = H(M)^d \bmod n$ . If Bob wants to verify whether  $S$  is a valid signature on message  $M$ , he simply checks whether  $S^3 = H(M) \bmod n$  holds.  $d$  is a private key known only to Alice and  $(n, 3)$  is a publicly known verification key that anyone can use to check if a message was signed using Alice's private signing key.

Suppose we instead used a **simplified** scheme for RSA signatures which skips using a hash function and instead uses  $M$  directly, so the signature  $S$  on a message  $M$  is  $S = M^d \bmod n$ . In other words, if Alice wants to send a signed message to Bob, she will send  $(M, S)$  to Bob where  $S = M^d \bmod n$  is computed using her private signing key  $d$ .

- (a) With this **simplified** RSA scheme, how can Bob verify whether  $S$  is a valid signature on message  $M$ ? In other words, what equation should he check, to confirm whether  $M$  was validly signed by Alice?

**Solution:**  $S^3 = M \bmod n$ .

- (b) Mallory learns that Alice and Bob are using the **simplified** signature scheme described above and decides to trick Bob into believing that one of Mallory's messages is from Alice. Explain how Mallory can find an  $(M, S)$  pair such that  $S$  will be a valid signature on  $M$ .

You should assume that Mallory knows Alice's public key  $n$ , but not Alice's private key  $d$ . The message  $M$  does not have to be chosen in advance and can be gibberish.

**Solution:** Mallory should choose some random value to be  $S$  and then compute  $S^3 \bmod n$  to find the corresponding  $M$  value. This  $(M, S)$  pair will satisfy the equation in part (a).

**Alternative solution:** Choose  $M = 1$  and  $S = 1$ . This will satisfy the equation.

- (c) Is the attack in part (b) possible against the **standard** RSA signature scheme (the one that includes the cryptographic hash function)? Why or why not?

**Solution:** This attack is not possible. A hash function is one way, so the attack in part (b) won't work: we can pick a random  $S$  and cube it, but then we'd need to find some message  $M$  such that  $H(M)$  is equal to this value, and that's not possible since  $H$  is one-way.

Comment: This is why the real RSA signature scheme includes a hash function: exactly to prevent the attack you've seen in this question.